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Skyrmions of Different Types

Tie-Zhong Li[1]

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We suggest two models of skyrmions. In one of them the vector mesons Z_μ^0 and $W_\mu^\pm$ may stabilize the skyrmion formed by Higgs bosons. In another the skyrmion is composed of three preons via the interaction with the Higgs bosons.

1. INTRODUCTION

The main results of our previous paper (Tie-Zhong Li, 1988) are as follows:

(i) We review Skyrme's Lagrangian from a different angle. The Lagrangian of the Skyrme model (Adkins *et al.*, 1983) is

$$L=\frac{1}{16}f^2\,\mathrm{Tr}(\partial_\mu U\cdot\partial^\mu U^+)+\frac{1}{32e^2}\,\mathrm{Tr}[(\partial_\mu U)U^+,(\partial_\nu U)U^+]^2 \tag{1}$$

where f is the decay constant of the pion, e is a dimensionless parameter, $U=\exp[(2i/f)\boldsymbol{\tau}\cdot\boldsymbol{\pi}]$, and $\boldsymbol{\pi}$ is the pseudoscalar triplet of the pion field. If we take away the physical meanings of the field and parameters from equation (1) and only consider equation (1) as a mathematical formula, then $\boldsymbol{\pi}$ can be not only a pion field, but also an arbitrary pseudoscalar triplet field.

(ii) We have found a pseudoscalar Higgs triplet **H** in a two-Higgs-doublet model and established the Lagrangian

$$L=\frac{1}{16}F^2\,\mathrm{Tr}(\partial_\mu U\cdot\partial^\mu U^+)+\frac{1}{32E^2}\,\mathrm{Tr}\{[(\partial_\mu U)U^+,(\partial_\nu U)U^+]^2\}$$
$$+\frac{1}{8}m_H^2F^2(\mathrm{Tr}\,U-2) \tag{2}$$

[1]Center of Theoretical Physics, CCAST (World Laboratory), and Institute of High Energy Physics, Academia Sinica, Beijing, China.

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where $U=\exp[(2i/F)\boldsymbol{\tau}\cdot\mathbf{H}]$. Equation (2) is chiral symmetric if we drop its third term. Then this Lagrangian is the same as equation (1) in mathematical form, but they are not the same in the field quantity, the strength of the interaction, and the parameters. However, the formation of the soliton does not depend on the strength of the interaction, it depends only on the character of the nonlinear interaction (Lee, 1981). The difference of the parameters has no influence on whether soliton solutions exist. So the existence of a skyrmion solution will be natural in the Lagrangian equation (2) without the third term. The skyrmion of this type possesses spin 1/2 and the topological charge 1.

(iii) We have explored naively the physical meaning of this skyrmion. We argue that the skyrmions of this type may be interpreted as leptons or quarks.

In this paper we shall extend this idea and discuss other models.

2. FIRST MODEL

We may substitute the interaction of the vector mesons for the second term of equation (2) in order to stabilize the soliton. For example, if we only consider the interaction of Z^0, then the Lagrangian

$$L=\tfrac{1}{16}f_H^2\,\mathrm{Tr}(\partial_\mu U\cdot\partial^\mu U^+)+\tfrac{1}{4}(\partial_\mu Z_\nu-\partial_\nu Z_\mu)(\partial^\mu Z^\nu-\partial_\nu Z^\mu)+\tfrac{1}{2}m_Z^2 Z_\mu Z^\mu$$
$$+\beta Z_\mu\cdot B^\mu+\tfrac{1}{8}f_H^2 M_H^2(\mathrm{Tr}\,U-2),\qquad Z_\mu\equiv Z_\mu^0 \tag{3}$$

may be used, where β is the Z_μ^0 decay constant, and B^μ is the current of the skyrmion[2] (fermion) that is formed by the Higgs triplet $\mathbf{H}$,

$$B_\mu=\left(\frac{\varepsilon^{\mu\nu\alpha\beta}}{24\pi^2}\right)\mathrm{Tr}[(U^+\partial_\nu U)(U^+\partial_\alpha U)(U^+\partial_\beta U)] \tag{4}$$

Substituting the Skyrme ansatz $U_0(x)=\exp[iF(r)\boldsymbol{\tau}\cdot\mathbf{r}]$ into equation (3), we get

$$F''(r)+\frac{1}{r}\left[F'(r)(2+\cos 2F(r))+\frac{\sin 2F(r)}{r}\right]$$
$$+\frac{2\beta}{\pi^2 f_H^2}\,\frac{\sin^2 F(r)}{r^2}\left[Z'(r)-\frac{2Z(r)}{r}\right]=0 \tag{5}$$
$$Z''(r)-\frac{2}{r}Z'(r)+m_Z^2 Z(r)-\frac{\beta}{2\pi^2}\,\frac{\sin^2 F(r)}{r^2}F'(r)=0$$

[2]If the skyrmion formed by the Higgs triplet $\mathbf{H}$ may be interpreted as a lepton or quark, then B_μ may be naturally interpreted as the lepton or quark current.

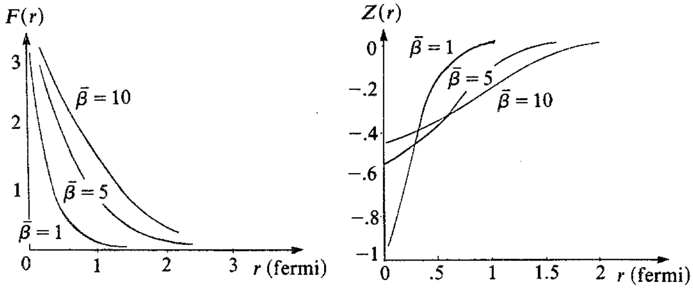

Fig. 1. Curves $F(r)$ and $Z(r)$ for different values of $\bar{\beta}=\beta m_Z/2\pi^2 f_H$.

Adkins and Nappi (1984) obtained a numerical solution of equations (5) for $\bar{\beta}=\beta m_Z/2\pi^2 f_H$. It is shown in Figure 1.[3] The $\bar{\beta}$ in their calculation is determined by the mass of the baryon (original skyrmion). The $\bar{\beta}$ here cannot be determined, because the mass of the skyrmion of this type is unknown. However, we may know from Figure 1 that the soliton exists within the variation of one order of magnitude of $\bar{\alpha}$, in other words, whether the existence of the soliton solution in (5) is not sensitive to the variation of the $\bar{\beta}$. So we may still expect that (5) has a Skyrmion solution. Actually, the only difference between the Lagrangian of Adkins and Nappi (1984) and equation (3) are field and parameters. However, these do not influence whether there exist soliton solutions (Lee, 1981). Exactly in the same way, if we substitute $W^{\pm}$ for Z^0_μ in (3), then there also exist soliton solutions stabilized by $W^{\pm}_\mu$. However, if we substitute A_μ or A^α_μ for Z^0_μ in (3), it cannot stabilize the soliton. Because the photon and gluon are massless, they cannot couple directly with the skyrmion (fermion) current in the way of Adkins and Nappi (1984). However, it is not impossible that the presence of A_μ or A^α_μ may stabilize the soliton in some other ways. The presence of Z^0_μ and $W^{\pm}_\mu$ only stabilizes the soliton which is formed by Higgs. However, it remains to be explored whether the soliton can be formed if there exist only vector mesons (Z^0_μ, $W^{\pm}_\mu$, A_μ, or A^α_μ) and no Higgs and whether we can still use the mathematical form of Skyrme's Lagrangian.

3. SECOND MODEL

As mentioned before, skyrmions do not include an interior structure. Now let us discuss another skyrmion including some interior structure. This skyrmion is composed of three preons (or subquarks) via the Higgs interaction. It will actually be a hybrid skyrmion model. The Higgs spectrum

[3]Figure 1 here is equal to Figures 1 and 2 and of Adkins and Nappi (1984) except for the meaning of the field and parameters.

will consist of the pseudoscalar triplet $\mathbf{H}$ (above-mentioned $h^{\pm 0}$) and a neutral scalar h_0 and the $\mathbf{H}$ may be written in a real basis as

$$\mathbf{H}=\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

So the Higgs Lagrangian may be written as[4]

$$L_0=\partial_\mu h_\alpha \cdot \partial^\mu h_\alpha + \partial_\mu h_0 \cdot \partial^\mu h_0 - \lambda^2\left[(h_\alpha h_\alpha + h_0 h_0)+\frac{\mu^2}{2\lambda}\right]+\frac{\mu^4}{4\lambda^2} \tag{6}$$

in preon space, $\alpha = 1, 2, 3$. Assume there exist two kinds of preon ψ_1 and ψ_2 that are fermions. The free preon satisfies the Dirac equation. The ψ_1 and ψ_2 are, in the fundamental representation of $SU(2)$,

$$\psi=\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{7}$$

The Lagrangian for the interacting Higgs-preon system can be written as

$$\begin{aligned} L=&\bar{\psi}(x)\cdot\{i\partial + g[h_0(x)+i\boldsymbol{\tau}\cdot\mathbf{H}(x)r_5]\}\psi(x) \\ &+\partial_\mu h_\alpha(x)\partial^\mu h_\alpha(x)+\partial_\mu h_0(x)\partial^\mu h_0(x) \\ &-\lambda^2\left\{[h_\alpha(x)h_\alpha(x)+h_0(x)h_0(x)]+\frac{\mu^2}{2\lambda}\right\}^2+\frac{\mu^4}{4\lambda^2} \end{aligned} \tag{8}$$

By solving equation (8), we obtain the Higgs and preon field density and we will know whether or not the soliton may be formed. Equation (8) forms a set of coupled nonlinear equations whose exact solution does not exist and analysis is complicated.

Another approach is to add explicitly a small symmetry-breaking term $h_0(x)$ for equation (8), similar to that in the σ-model of Gell-Mann and Levy (1960)

$$\begin{aligned} L=&\bar{\psi}(x)\cdot\{i\partial + g[h_0(x)+i\boldsymbol{\tau}\cdot\mathbf{H}(x)r_5]\}\psi(x) \\ &+\partial_\mu h_\alpha(x)\cdot\partial^\mu h_\alpha(x)+\partial_\mu h_0(x) \\ &\times\partial^\mu h_0(x)-\lambda^2[h_0^2(x)+h_\alpha^2(x)-\nu^2]^2-v m_{\mathrm{H}}^2 h_0(x) \\ &-\left(v^2 m_{\mathrm{H}}^2-\frac{1}{4}\frac{m_{\mathrm{H}}^2}{\lambda^2}\right) \end{aligned} \tag{9}$$

[4]Here $\bar{H}$ and φ_0 share the same masses and coupling constant. This will be relaxed if necessary in the following discussion.

where we have changed some free parameters and made some modification for the parameters of **H** and h_0 in order to use known results and techniques (Birse and Banerjee, 1984, 1985). It is remarkable that equation (9) corresponds to the Lagrangian of the Birse and Banerjee (1984, 1985) model (BB model). The $\psi(x)$, $\mathbf{H}(x)$, and $h_0(x)$ describe the quarks, pion, and sigma field in the BB model. Though the meaning of the parameters is not quite the same, solving the field equations directly from (9) is almost the same as solving that of the BB model. The following discussion of (9) closely parallels that of the BB model [see Birse and Banerjee (1984, 1985) for a careful treatment; We describe only the main content].

The vacuum of equation (9) is spontaneously broken, $\langle h_0 \rangle = -v$ and $\langle \mathbf{H} \rangle = 0$, in the absence of the preon source term because the sign of the quadratic scalar term has been chosen to be negative. However, it does not produce a Goldstone boson, since the linear term $h_0(x)$ has emerged which just produces the physical Higgs mass $m_{\mathbf{H}}$. The only two free parameters g and λ are reexpressed in terms of the preon and the Higgs masses (λ and g are dimensionless parameters)

$$\begin{aligned} m_{pr} &= gv \\ m_{h_0}^2 &= 2\lambda^2 v^2 + m_{\mathbf{H}}^2 \\ \nu^2 &= v^2 - m_{\mathbf{H}}^2/\lambda^2 \end{aligned} \tag{10}$$

The time-independent c-number fields $h_0(r)$ and $\mathbf{H}(r)$ satisfy the equations

$$\begin{aligned} &\frac{d^2 h_0(r)}{dr^2} + \frac{2}{r}\frac{dh_0(r)}{dr} - \lambda^2[h_0^2(r) + H^2(r) - \nu^2]h_0(r) \\ &\quad + 3g[G^2 - F^2(r)] - vm_{\mathbf{H}}^2 = 0 \\ &\frac{d^2 H(r)}{dr^2} + \frac{2}{r}\frac{dH(r)}{dr} - \frac{2}{r^2}H(r) - \lambda^2[H^2(r) + h_0^2(r) \\ &\quad - \nu^2]H(r) - 6gG(r)F(r) = 0 \end{aligned} \tag{11}$$

The preon wave function satisfies the Dirac eigenvalue equations,

$$\begin{aligned} \frac{dG(r)}{dr} + g[H(r) + h_0(r)F(r)] &= \omega F(r) \\ \frac{dF(r)}{dr} + \frac{5}{3}\frac{1}{\gamma}F(r) - g[H(r)F(r) - h_0(r)G(r)] &= -\omega G(r) \end{aligned} \tag{12}$$

The normalized Dirac spinor state of the preon has been used in the process of obtaining equations (11) and (12),

$$P(r)=\begin{pmatrix} G(r)X_h \\ -iF(r)\boldsymbol{\sigma}\cdot\mathbf{r}X_h \end{pmatrix} \tag{13}$$

According to the idea of the hedgehog soliton (skyrmion), the vector Higgs field **H** in the above equations should be in a pure p wave, i.e.,

$$H_\alpha(r)=H(r)\mathbf{r}_\alpha \tag{14}$$

and a set of boundary conditions should be imposed on the solutions of equations (11) and (12):

$$\begin{aligned} h_0(r) &\xrightarrow[r\to\infty]{} -v, \qquad H(r)\xrightarrow[r\to\infty]{} 0 \\ G(r) &\xrightarrow[r\to\infty]{} 0, \qquad F(r)\xrightarrow[r\to\infty]{} 0 \end{aligned} \tag{15}$$

However, there do not exist solutions for equations (11) and (12), so we can only solve them numerically. We should choose some satisfactory numerical value (satisfactory parameters) for $m_{\mathbf{H}}$, m_{h_0}, m_{pr}, g, and λ, then calculate the soliton solutions of the field from equations (11) and (12). However, satisfactory parameters are not known. It is not possible at present to look for solutions of satisfactory parameters, so equations (11) and (12) need to be aside until such parameters are known.

However, we can hope to get some information about whether there exist solitons by using some parameters. We first make a simple treatment of equations (11) and (12). For example, as a numerical exercise, because equations (11) and (12) are mathematically equivalent to equations (5) and (6) of the BB model, we only need to change their mass unit (MeV) into a new unit (MeV$\times c$) relevant to our purpose, $c>0$. Then we can obtain directly the following numerical solution of the Higgs and preon density of our equations (11) and (12) using known results (Birse and Banerjee, 1984). These numerical solutions are shown in Figure 2.[5]

(i) The Higgs fields h_0 and **H** may form solitons.

(ii) The soliton of the Higgs field in our model is similar to the soliton first studied by Skyrme (1961). But the winding number is associated with the BB model (Birse and Banerjee, 1984). The soliton in our model does not contribute a fermion number (baryon number). So the fermion number of the soliton may be given only the preons.

(iii) We may see from Figure 2 that both vector Higgs **H** and scalar h_0 have an attractive interaction with preons. So our soliton with preons localized near the center is stable.

(iv) The preon rms radius is $0.7c^{-1}$ fm.

[5]Figure 2 here is equal to Figure 1 of the BB model except for the meaning of the curves and their units.

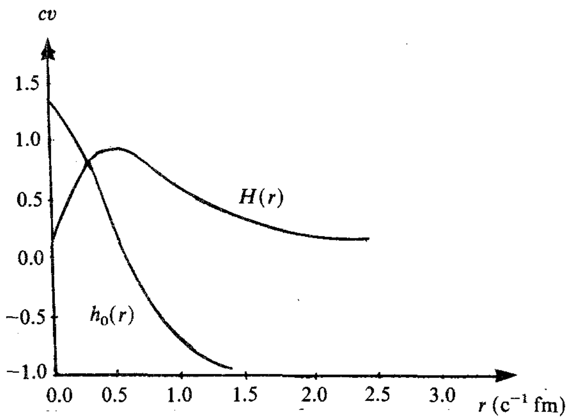

Fig. 2. Curves $H(r)$ and $h_0(r)$ in units of cv.

The above-mentioned results show that there exist solutions corresponding to solitons in this model. In other words, we get a stable configuration of preons forming the hedgehog soliton, but it depends on the selected parameters.

4. SUMMARY

In this paper we suggest two models based on Tie-Zhong Li (1988):

(i) The interaction of the vector mesons Z^0_μ and $W^\pm_\mu$ with the current of the skyrmion may stabilize the skyrmion that is formed by the Higgs bosons.

(ii) Another skyrmion may include an interior structure that is composed of three preons via the interaction of the Higgs bosons. However, we only get a set of equations that describe it, exact numerical solutions need further effort. We get the solution of the skyrmion under some parameters that are not realistic.

The physical meaning of the skyrmions of these two types is similar to that of Tie-Zhong Li (1988) and perhaps they may be interpreted as leptons or quarks.

Gipson and Tze (1981) found a possible heavy soliton, but they must use a heavy and strongly coupled Higgs. Soni *et al* (1987, 1989) have discussed the strongly Yukawa-coupled model in a Skyrme Higgs background. The Higgs must not be heavy and strongly coupled in order to get the skyrmion.

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